

Appendix B: TouchMath Basics

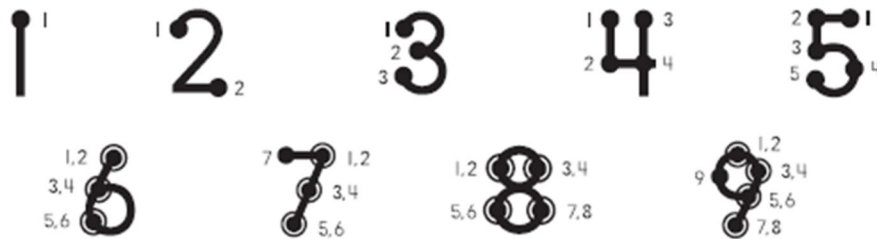
(adapted from 2012 [TouchMath Teacher Training Manual](#))



Counting Touchpoints:

Each TouchMath numeral has a corresponding number of TouchPoints placed upon the numeral.

It is important to use the same pattern consistently. The single TouchPoints are touched and counted once, while the double TouchPoints are touched and counted twice. The correct Touching/Counting patterns are shown below:



Zero has no TouchPoints and no value. Never touch and count the zero. The zero is also used as a placeholder in place value.

Beginning Addition using TouchMath:

In the beginning you will count all the TouchPoints on both numbers.

Teach the phrase, “I touch and count all the TouchPoints on the numbers.”

1. Begin counting with the top number in the column, 5: “1,2,3,4,5.”
2. Continue counting on the 4: “6,7,8,9”. Record the answer: 9.
3. Reinforce the addition facts by repeating the problem and answer aloud, “Five plus four equals 9.”

$$\begin{array}{r} 5 \\ + 4 \\ \hline 9 \end{array}$$

Addition with Counting On:

When a student can identify the greater number, you can introduce the concept of "Counting On". This allows you to start with the greater number and then continue counting.

Teach the Addition Counting On statement, "I touch the greater number, say its name and continue counting on the TouchPoints of the other number(s)."

1. Touch the greater number and say its name, "8".
2. Continue counting on the TouchPoints of the 4: "9,10,11,12". Record the answer 12.
3. Reinforce the addition facts by repeating the problem and answer aloud. "Eight plus Four equals Twelve"

Note the greater number may not have TouchPoints on the worksheets to help the students identify the greater number. If the student still needs all the TouchPoints, you can draw them in.

$$\begin{array}{r} 8 \\ + 4 \\ \hline 12 \end{array}$$

Addition Without Regrouping

Teach the Arrow Statement. "I start on the side with the arrow. The arrow is in the ones column on the right side."

1. Say the greater number in the ones column: "6," and continue counting on the TouchPoints of the 2: "7,8."
2. Record the answer: 8.
3. Repeat step 2 in the tens column: "4" and continue counting on the TouchPoints of the 3: "5,6,7." Record the answer 7.
4. Repeat the problem and answer aloud. "forty-two plus thirty-six equals seventy-eight."

Tens	Ones
↓	↓
4	2
+	3
3	6
78	

Addition With Regrouping

Teach the Addition Regrouping statement, “I must regroup if my answer is greater than 9”.
Begin in the one’s column below the arrow.

1. Say the name of the greater number “8” and continue counting on the 7: “9,10,11,12,13,14,15”.
2. Use the box to record the number of the tens regrouped to the tens column (1).
3. Record the number of ones (5). Add the tens column starting at the top “1,2,3,4,5,6”. Record the answer 6.
4. Repeat the problem and answer aloud. “twenty-seven plus thirty-eight equals sixty-five”.

Tens	Ones
□	↓
2	7
+	3
3	8
65	

Subtraction – Counting Backward

Students will need to count backward from 18 and every other number below 18. Practice counting backward from any number and stopping at any number. When teaching backward counting work on one number at a time and practice until students are proficient with counting backward from that number.



Beginning Subtraction

Teach the Beginning Subtraction Statement, “I touch the first number, say its name and count backward on the TouchPoints of the other number.”

1. Touch the top number and say its name: “7”.
2. Count backward on the TouchPoints of the 5: “6,5,4,3,2.”

- Record the answer: 2.
- Reinforce the subtraction facts by repeating the problem and the answer aloud. "seven minus five equals two"

$$\begin{array}{r} 7 \\ - 5 \\ \hline 2 \end{array}$$

Subtraction Without Regrouping

Teach double-digit subtraction without regrouping by starting with the Arrow Statement, "I start on the side with the arrow. The arrow is in the ones column on the right side."

- Touch the top number and say its name: "7".
- Count backward on the TouchPoints of the 4: "6,5,4,3"
- Record the answer 3.
- Repeat the process in the tens column.
- Reinforce the subtraction facts by repeating the problem and answer aloud. "sixty-seven minus twenty-four equals forty-three".

Tens	Ones
	↓
6	7
- 2	4
4	3

Subtraction With Regrouping

Teach the Subtraction Regrouping Statement: "If I cannot count all of the TouchPoints, I must regroup."


- Count backwards in the ones column from 3: "2,1,0". Students will discover they cannot count all the TouchPoints on the 6.
- Regroup or borrow one ten from the tens column.
- Cross out the top number, 8, and write the regrouped number, 7, on the bar.
- Write the regrouped ten in front of the 3 in the ones column, making it as large as the three changing the number to 13.
- Count backward from the regrouped number, 13: "12,11, 10, 9, 8, 7."

6. Record the answer: 7.
7. Subtract the tens column first say the regrouped number 7, then "6,5,4." Record the answer 4.
8. Repeat the subtraction problem and answer aloud. "eighty-three minus thirty-six equals forty-seven".

$$\begin{array}{r}
 \text{Tens} \quad \text{Ones} \\
 \underline{7} \quad \downarrow \\
 83 \\
 - 36 \\
 \hline
 47
 \end{array}$$

Multiplication and Division - Skip Counting

Skip counting is a critical skill for multiplication, division and higher mathematical development. Teach one sequence at a time. Start with the 2s sequence: 2,4,6....to20. Teach each sequence for the 2s through 9s.

$$\begin{array}{l}
 5 = \square \\
 5 + 5 = \square \\
 5 + 5 + 5 = \square \\
 5 + 5 + 5 + 5 = \square \\
 5 + 5 + 5 + 5 + 5 = \square \\
 5 + 5 + 5 + 5 + 5 + 5 = \square \\
 5 + 5 + 5 + 5 + 5 + 5 + 5 = \square \\
 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = \square \\
 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = \square
 \end{array}$$


Beginning Multiplication

Teach the Multiplication statement: "I skip count by one number while touching the TouchPoints on the other number."

4. Skip count by 2 while touching the TouchPoints on the 6: "2,4,6,8,10,12".
5. Record the answer: 12.
6. Reinforce the facts by repeating the problem and answer aloud. "six times two equals twelve"

$$\begin{array}{r}
 6 \\
 \times 2 \\
 \hline
 12
 \end{array}$$

Multiplication Without Regrouping

Teach the statement: "I start on the side with the arrow. The arrow is on the right side."

6. Skip count by 3 while touching the TouchPoints on the 2: "3,6".

7. Record the answer: 6.
8. Skip count while touching the TouchPoints on the 3: "3,6,9".
9. Record the answer: 9.
10. Reinforce the numbers by saying the problem and answer aloud. "thirty-two times 3 equals 96."

Tens	Ones
↓	↓
3	2
×	3
<div style="display: flex; justify-content: space-around; width: 100%;"> 9 6 </div>	

Multiplication with Regrouping

Begin by teaching the regrouping statement: "I must regroup if my answer is greater than 9."

6. Skip count by 8 while touching the TouchPoints of the 7: "8,16,24,32,40,56".
7. Write the 5 tens in the box first, then write the 6 ones in the ones column.
8. Count by 8 while touching the TouchPoints on the 6: "8,16,24,32,40,48".
9. Add the tens in the box: 49,50,51,52,53.
10. Record the answer 53.

Tens	Ones
↓	↓
5	7
6	7
×	8
<div style="display: flex; justify-content: space-around; width: 100%;"> 5 3 6 </div>	

Beginning Division

Teach the division statement: "I skip count by the divisor and get as close to the dividend as possible without going over the dividend."

5. Skip count by 2: "2,4,6,8".
6. As each number is counted make a tally mark in the box above the division problem.
7. Count the tally marks.
8. Record the answer: 4.

$$\begin{array}{r} \boxed{///} \\ 4 \\ \hline 2 \overline{) 8} \end{array}$$

Division with Remainders

Teach the division with remainders statement: “I skip count by the divisor and get as close to the dividend as possible without going over the dividend. Then I continue counting by one up to the dividend.”

4. Skip count by 3. As each number is counted, make a tally mark in the box. Come as close as you can without going over: “3,6,9,12,15,18,21.”
5. To find the remainder, continue counting forward by ones. Put dots outside of the box for each number counted: “22,23.”
6. Count the tally marks and record the answer: 7. Count the dots and record the remainder: 2.

$$\begin{array}{r} \boxed{/// //} \bullet \bullet \\ 7r2 \\ \hline 3 \overline{) 23} \end{array}$$

Long Division

12. Divide the 4 in the hundreds column by 3, and record the answer.
13. Multiply 3 by 1 and record the answer below the 4.
14. Subtract 3 from 4 and record the answer.
15. Bring down the 6 in the tens column.
16. Divide 16 by 3 and record the answer.
17. Multiply 3 by 5 and record the answer.
18. Subtract 15 from 16 and record the answer.
19. Bring down the 0 in the ones column.
20. Divide 10 by 3 and record the answer.
21. Multiply 3 by 3 and record the answer.
22. Subtract 9 from 10 and record the answer, which is the remainder.

$$\begin{array}{r}
 3 \overline{)460} \\
 \underline{-3} \\
 16 \\
 \underline{-15} \\
 10 \\
 \underline{-9} \\
 10
 \end{array}$$

The diagram shows the long division of 460 by 3. A horizontal line is above the 460. A vertical line is to the left of the 460. A horizontal line is below the 460. A vertical line is to the right of the 460. The 3 is to the left of the vertical line. The 4 is in the hundreds column, the 6 is in the tens column, and the 0 is in the ones column. There are three subtraction lines: one under the 4, one under the 16, and one under the 10. There are three arrows: one pointing down from the 4 to the 1, one pointing down from the 6 to the 5, and one pointing down from the 0 to the 0.

Short Division

6. Divide the 4 in the hundreds column by 3 and record the answer: 1.
7. Mentally subtract 3 from 4 and write the 1 in front of the 6 in the tens column.
8. Divide 16 by 3. Write the answer 5.
9. Mentally subtract 15 from 16. Write the 1 in front of the 0 in the ones column.
10. Divide 10 by 3. Write the answer: 3. Then write the remainder: 1

$$\begin{array}{r}
 153 \text{ r } 1 \\
 3 \overline{)460} \\
 \underline{3} \\
 16 \\
 \underline{15} \\
 10 \\
 \underline{9} \\
 10
 \end{array}$$

The diagram shows the short division of 460 by 3. The answer 153 and remainder 1 are written above the 460. The 3 is to the left of the 460. The 4 is in the hundreds column, the 6 is in the tens column, and the 0 is in the ones column. The 1 is in the hundreds column, the 5 is in the tens column, and the 3 is in the ones column. The remainder 1 is to the right of the 0. There are three subtraction lines: one under the 4, one under the 16, and one under the 10.